Int. J. Multiphase Flow Vol. 11, No. 4, pp. 565–569, 1985
Printed in the U.S.A.

0301-9322/85 $3.00 + .00

BRIEF COMMUNICATION

ON ASYMPTOTIC RESULTS FOR FILM POOL BOILING ON VERTICAL SURFACES

AKIRA NAKAYAMA and HITOSHI KOYAMA
Department of Mechanical Engineering, Faculty of Engineering, Shizuoka University, Hamamatsu, 432 Japan

(*Received* 24 *July* 1984; *in revised form* 25 *November* 1984)

INTRODUCTION

Koh (1962) viewed film pool boiling as a two phase boundary layer problem, and solved the full boundary layer equations using a similarity transformation. The similarity transformation gives a set of ordinary differential equations which can be solved numerically. A limited number of numerical integration results have been provided by Koh and Nishikawa & Ito (1966) for different sets of the parameters.

In order to investigate combined effects of the parameters involved in a particular heat transfer problem, it is often more advantageous to employ an integral formulation than the numerical calculations. A simple method based on the von Kármán integral relation is suggested to give asymptotic results for the film pool boiling in a saturated liquid.

ANALYSIS

A physical model under consideration is depicted in figure 1. A vertical flat plate of constant temperature T_w is placed in a large volume of saturated liquid of temperature T_s. The degree of superheating is moderate, and the vapor film is thin, so that the inertia and convective terms in the vapor boundary layer equations can be dropped. The resulting equations in the boundary coordinates (x, y) are

$$\nu \frac{\partial^2 u}{\partial y^2} + g' = 0, \qquad [1a]$$

$$\frac{\partial^2 T}{\partial y^2} = 0, \qquad [1b]$$

with

$$g' \equiv g(\rho_L - \rho)/\rho, \qquad [1c]$$

where u is the streamwise velocity component; T, the temperature; ν, the kinematic viscosity; ρ, the density; and g, the acceleration of gravity. The subscript L refers to the liquid while no subscript is assigned for the vapor. The integration of [1a] and [1b] yields

$$u/u_i = (1 + \Lambda)\eta - \Lambda\eta^2, \qquad [2a]$$

and

$$(T - T_s)/(T_w - T_s) = 1 - \eta, \qquad [2b]$$

with

$$\Lambda \equiv g'\delta^2/2\nu u_i, \qquad [2c]$$

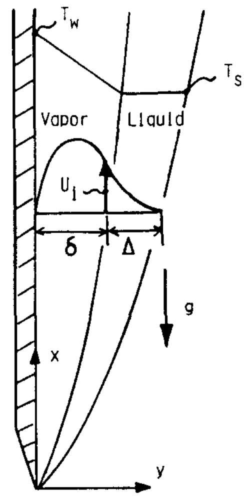

Figure 1. Physical model and coordinate system.

where δ is the vapor film thickness and u_i is the interfacial velocity.

An energy balance consideration at the liquid-vapor interface leads to

$$h_{LG}\frac{\mathrm{d}}{\mathrm{d}x}\int_0^{\delta}\rho u \mathrm{d}y = -k\frac{\partial T}{\partial y}\Big|_{y=\delta}, \qquad [3]$$

where h_{LG} is the latent heat of vaporization.

After substituting [2], [3] can be transformed into an ordinary differential equation in terms of δ^4, which can readily be integrated to give

$$(\delta/x)^4\,(\mathrm{Gr}_x\,\mathrm{Pr}/H) = \frac{16\Lambda}{\Lambda + 3}. \qquad [4a]$$

Combining [4a] and [2c],

$$(u_i^2/g'x)\,(\mathrm{Pr}/H) = \frac{4}{\Lambda(\Lambda + 3)}, \qquad [4b]$$

where

$$H \equiv C_p(T_w - T_s)/h_{LG} \qquad [4c]$$

$$\mathrm{Gr}_x \equiv g'x^3/\nu^2, \qquad [4d]$$

and C_p is the specific heat of vapor, and Pr, the Prandtl number of vapor.

Now, for the liquid boundary layer within $\delta \leq y \leq \delta + \Delta$, the von Kármán integral relation can be written as:

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_{\delta}^{\delta+\Delta}\rho_L u^2\,\mathrm{d}y + u_i\frac{\mathrm{d}}{\mathrm{d}x}\int_0^{\delta}\rho u\,\mathrm{d}y = -\mu_L\frac{\partial u}{\partial y}\Big|_{y=\delta}. \qquad [5a]$$

The second term in the left hand side of [5a] representing the momentum sink to the vapor film is of the order (ρ/ρ_L) and is usually negligible when compared with the first term. Therefore,

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_{\delta}^{\delta+\Delta}\rho_L u^2\,\mathrm{d}y = -\mu_L\frac{\partial u}{\partial y}\Big|_{y=\delta}. \qquad [5b]$$

It is interesting to note that a similar consideration was given by Chen (1961) and Shekriladze & Gomelauri (1966) for the saturated film condensation, which led them to neglect the first term of [5a] (note, the roles of liquid and vapor are interchanged).

The following velocity profile, which closely represents the exact solution (Koh 1962), is assumed for the liquid layer:

$$u/u_i = (1 - \eta_L)^2, \tag{6a}$$

where

$$\eta_L \equiv (y - \delta)/\Delta. \tag{6b}$$

Further, with an aid of [4b], [5b] can be solved as

$$(\Delta/x)^4\,(\mathrm{Gr}_x\,H/\mathrm{Pr})\,(\nu/\nu_L)^2 = 16\Lambda\,(\Lambda + 3). \tag{7}$$

The matching condition for the interfacial shear is given by

$$\mu\frac{\partial u}{\partial y}\Big|_{y=\delta} = \mu_L\frac{\partial u}{\partial y}\Big|_{y=\delta} \tag{8a}$$

or

$$\Delta/\delta = \left(\frac{2}{\Lambda - 1}\right)(\mu_L/\mu). \tag{8b}$$

The substitution of [4a] and [7] into [8b] yields the following characteristic equation:

$$\frac{H}{\mathrm{Pr}R} = \frac{1}{4}(\Lambda - 1)^2\,(\Lambda + 3), \tag{9a}$$

where

$$R \equiv \rho\mu/(\rho\mu)_L. \tag{9b}$$

Equation [9b] indicates that the solution of the problem depends solely on a single lumped parameter, $H/\mathrm{Pr}R$, when the vapor film is thin. Once the shape factor Λ is determined for a given $H/\mathrm{Pr}R$, the local Nusselt number $\mathrm{Nu}_x = (x/(T_s - T_w))\,(\partial T/\partial y)|_{y=0} = x/\delta$ may be calculated from

$$\mathrm{Nu}_x/(\mathrm{Gr}_x\mathrm{Pr}/H)^{1/4} = \frac{1}{2}\left(\frac{\Lambda + 3}{\Lambda}\right)^{1/4} \tag{10}$$

The effect of the thermal radiation may become significant when the plate is heated to a high temperature. Lubin (1969) introduced an elegant treatment to include radiation in his integral analysis with zero interfacial shear, which eventually led him to the formula identical to the one proposed by Bromley (1950). Equation [3], when the radiative contribution added to the right hand side term, reveals that the similarity solution is no longer possible for nonzero interfacial shear. However, the qualitative derivation made by Bromley may hold even for the present case of nonzero interfacial shear. Thus, the following formula is suggested for the practical evaluation of the total heat transfer coefficient h:

$$h = (h_c/h)^{1/3}h_c + h_r \tag{11}$$

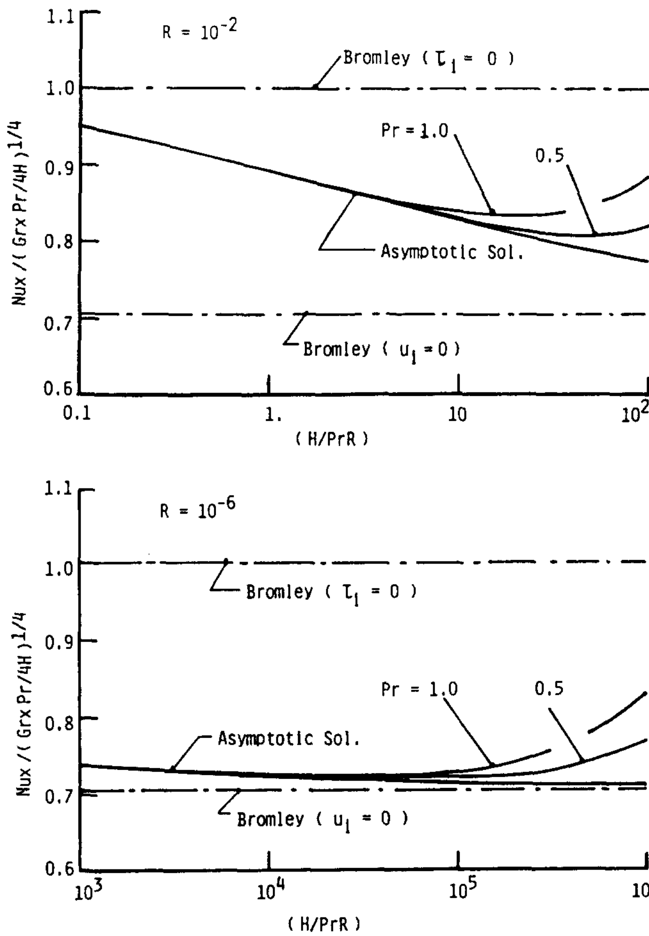

Figure 2. Comparison of asymptotic results and exact solutions (*a*) $R = 10^{-2}$; (*b*) $R = 10^{-6}$.

where $h_c = \mathrm{Nu}_x\,(k/x)$ as given by [10], and h_r is an equivalent heat transfer coefficient due to thermal radiation (Lubin 1969).

RESULTS

The asymptotic results given by [10] and [9a] have been derived under the condition of negligible inertia and convection within the vapor film. It can readily be shown that this condition is satisfied when $g'x/u_i^2$ is much greater than unity, or simply $H/Pr \ll 1$ by virtue of [4b] (note, $\Lambda \geq 1$ for a pool boiling). With this in mind, [10] along with [9a] is plotted for $R = 10^{-2}$ and 10^{-6} in figures 2 where the ordinate variable is chosen such that a direct comparison with the Bromley's solutions is possible. Koh's exact solutions are also illustrated in the figures. As may be expected, the curves of the exact solutions for different Pr overlap onto the present asymptotic curve as $H/\mathrm{Pr}R$ becomes sufficiently small.

Equation [9a] indicates $\Lambda \to 1$ as $H/\mathrm{Pr}R$ becomes small. As seen from [10], this limiting case corresponds to the Bromley's solution for zero interfacial shear. The other limiting case implicit in [10] and [9a] is the Bromley's solution for zero interfacial velocity, namely, $\mathrm{Nu}_x/(\mathrm{Gr}_x\mathrm{Pr}/H)^{1/4} = \frac{1}{2}$ as $\Lambda \gg 1$ or $H/\mathrm{Pr} \gg R$. Therefore, this Bromley's solution gives a fair heat transfer level only within the range of $R \ll H/\mathrm{Pr} \ll 1$, as observed in figure 2 (b).

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